

The purpose of this application note is to explain what Cp and Cpk are and how these statistical tools can be used to prevent shipping marginal products. Most people have at least heard of Cp or Cpk, but few people really understand the concepts behind the calculations. This is valuable information not just for Quality Managers, but also for the Engineers who design new products, as well as Purchasing Agents who have to maintain a list of qualified vendors. In fact the 1994 revision of ISO 9000 requires suppliers to evaluate their subcontractors on the basis of their ability to meet subcontract requirements (Section 4.6.2).

Under the method of quality acceptance that is currently used by most organizations (including the U.S. government) the customer determines the requirements for an item and sets up specified limits within which an item must fall to be acceptable. Anything falling within the specified limits is considered good; anything falling outside the limit is bad. This is the go/no-go approach that Genichi Taguchi calls the “goalpost” mentality. This system is illustrated in Figure 1: Part A is in the center of the specification range, part B is just inside the specification limit, part C is just outside the specification limit, and part D does not even come close to the specified range. In this system parts A and B are both considered equally good parts, Parts C and D are both considered equally bad parts. With this approach there is no measure of goodness and product of marginal quality is routinely accepted.

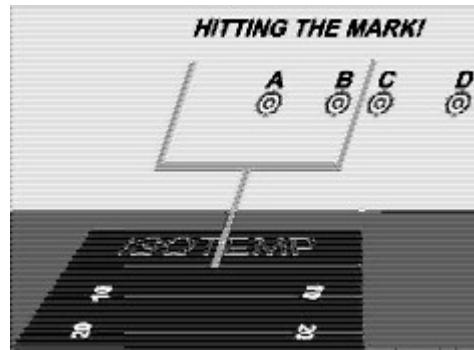


Figure 1

Organizations must stop using the “goalpost” approach and instead use a procurement process that defines quality requirements in terms of the distribution about the target value. Taguchi has stated this idea in terms of the loss function. The loss function essentially says that the further a product is from the target value, the greater the quality problems encountered by customers. This is where the process capability index, or Cp comes in to play. Cp is a measure of a process’s ability to meet a specification (or it is a measure of a group of items’ nearness to the target value). The formula for calculating Cp is as follows:

$$C_p = \frac{(\text{upper specification limit} - \text{lower specification limit})}{6\sigma}$$

Where  $\sigma$  is the symbol for **STANDARD DEVIATION**, multiplying by six gives the total process variation of 6 $\sigma$  (Six Sigma), which translates to  $\pm 3$  **STANDARD DEVIATIONS** from the mean. For a process with a normal bell-shaped distribution, which is symmetrical and centered, approximately 99.7 % of the values will be within  $\pm 3\sigma$ . The specification spread or total tolerance is the upper specification limit  $U$  minus the lower specification limit  $L$ .

The Cpk index is the position of the total process variation (6σ) in relation to the specification mean. The Cpk associated with a process or a group of items is either the value for Cpu or Cpl, whichever calculation results in the smallest number. The Cpu index is the position of the total process variation (6σ) in relation to the upper specification limit. The Cpl index is the position of the total process variation (6σ) in relation to the lower specification limit. Cpu and Cpl are defined with the following equations:

$$C_{pu} = \frac{(\text{upper specification} - \bar{X})}{3\sigma} \quad C_{pl} = \frac{(\bar{X} - \text{lower specification})}{3\sigma}$$

Where  $\bar{X}$  is the mean of the sample. (See Figure 2)

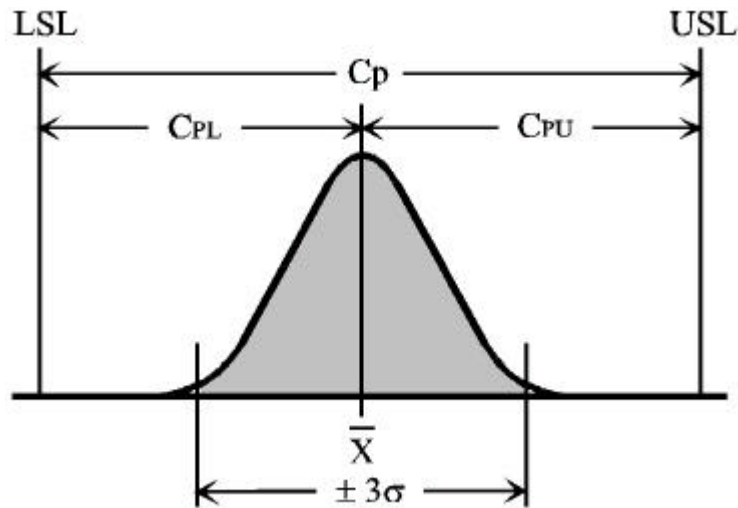


Figure 2

Examination of the equations for Cp, Cpk, Cpu, and Cpl yields the following:

$$C_{pu} = C_{pl} = C_{pk} = C_p$$

(Only when the process is centered about the target value.)

The purpose of a process capability study is to separate the inherent **RANDOM VARIABILITY** which will always be present, from the **ASSIGNABLE CAUSES** which must be investigated. TABLE 1, compares three processes that are centered within the specification limits versus three processes which are shifted ±1.5 sigma (σ) from the specification mean. (NOTE: A ±1.5 sigma shift from nominal is being considered the worst case shift for a normal bell curve distribution.)<sup>1</sup>

<sup>1</sup> This shift is a compensatory measure needed due to the cumulative effects of variation in the sample means calculated over a large number of groups.

TABLE 1 (Expected defects listed for six processes with Cp values ranging from 1.00 to 2.00)

Distribution	Cp	Centered	Cpk	Expected Defects
±3.0 sigma	1.00	NO	0.50	66,810 ppm
±3.0 sigma	1.00	YES	1.00	2,700 ppm
±4.5 sigma	1.50	NO	1.00	2,700 ppm
±4.5 sigma	1.50	YES	1.50	3.4 ppm
±6.0 sigma	2.00	NO	1.50	3.4 ppm
±6.0 sigma	2.00	YES	2.00	0.002 ppm

All of the major concepts of process capability can be explained by studying the values listed in TABLE 1. These concepts include the following:

- ? A process must be designed with a Cp value that is higher than the required Cpk value. (This accounts for the process not being perfectly centered within the specification limits.)
- ? The Cpk of a process can be affected in production by adjusting the centering of the process, but the Cp of a process will remain the same unless there is a definite change made in the process itself.
- ? A process which is not centered about the specification mean will have many more expected defects than a process which is relatively centered. (This difference may be as large as several orders of magnitude when the process is shifted ±1.5 sigma.)
- ? In order to ensure that a product or process has “Six Sigma” capability, the process must have a Cp ? 2.0. (This is to guarantee a Cpk ? 1.5 even if the process is not centered.)

A process capability study involves taking periodic samples from the process under controlled conditions and determining the sample mean and **STANDARD DEVIATION**. At this point, calculating the values for Cp and Cpk will tell you what the yield for the process should be minus the effects of any **ASSIGNABLE CAUSES**.

Comparing the actual yield for a process to the expected yield will show how much opportunity for improvement exists in the process. The key to the identification and removal of these **ASSIGNABLE CAUSES** is careful documentation of the process and any changes that are made to it. Probably the best way to begin reducing the total error present in the process is by listing all sources of error on a Pareto chart starting with those that have the greatest influence on the process. By ranking the sources of error you will be able to achieve the largest amount of improvement by concentrating on those problems which are most vital. In Pareto terms, it is the “vital few” (20 percent) of the variables that account for 80 percent of the total process error. Whereas the other sources of error are among the “trivial many” (80 percent) of the variables that make up only 20 percent of the total process error.

When designing a new product or process the goal is to ensure that the process is capable of consistently producing products that meet or exceed the specification requirements. The emphasis is on defect prevention, not detection. The benefits for the manufacturer of designing a highly capable process include higher yields, fewer defects, lower rework cost, and less inspection time. The benefits to the customer include better incoming quality levels, less inspection time, and the ability to implement dock-to-stock or just-in-time (JIT) delivery procedures. If a process has “Six Sigma” capability, excluding any **ASSIGNABLE CAUSES**, the expected defect rate will be ? 3.4 parts per million.

## Glossary

### Standard Deviation

Standard deviation is denoted by the Greek symbol Sigma ( $\sigma$ ) and is computed by the following formula:

$$\sigma = \sqrt{[(n)(\sum X^2) - (\sum X)^2] / [(n)(n-1)]}$$

Where X is the individual values for a sample of size n.

### Random Variability

Variations in data which result from causes which cannot be pinpointed or controlled. For these random variations a change in the source variables will not produce a highly predictable change in the response, e.g., a correlation does not exist. Includes inherent natural sources of variation.

### Assignable Causes

Variations in data which can be attributed to specific causes. For these non-random variations a change in the source variables will produce a significant change in the response, e.g., a correlation exists. The change may be due to an intermittent in-phase effect or a constant cause system which may or may not be highly predictable. An assignable cause is often signaled by an excessive number of data points outside a control limit and/or a non-random pattern within the control limits.

## References

- Windham, Jeff, "Implementing Deming's Fourth Point," *Quality Progress*, December 1995, pp. 43-48.  
Genichi Taguchi, *Introduction to Quality Engineering* (Dearborn MI: American Supplier Institute, Inc., 1986).  
Hradesky, John L., *Total Quality Management Handbook* (New York, NY: McGraw-Hill Inc., 1995).  
Discrete and Materials Technologies Group, *Reliability Audit Report First Quarter 1991* (Motorola Inc., 1991).  
Harry, Mikel J., *Six Sigma Producibility Analysis and Process Characterization* (Motorola Inc., 1992).

For reprints of this article, ask for document number 146-008.



P.O. Box 3389  
Charlottesville, VA 22903 USA  
Voice: (434) 295-3101  
FAX: (434) 977-1849  
www.isotemp.com

This application note was written in May 1996 by David L. Ruffner.  
Isotemp Research Inc. is an American company building performance ovens and oscillators since 1968