APPLICATION NOTE

OSCILLATOR SHORT TERM STABILITY: (root) Allan Variance

Normally, when trying to determine how tightly grouped a set of data is, the standard deviation (σ, or square root of variance, σ^2) of that data is calculated. However, when making this calculation on a group of frequency measurements taken from an oscillator, long term stability variables (such as aging and ambient effects) can begin to obscure the short term stability of that oscillator. In this instance a calculation called Allan variance (named after David Allan of NIST) provides better results. Simply put, instead of calculating the variance for an entire set of data at once, the variances from progressing pairs of data points are averaged, minimizing non-short term factors.

In practice, root Allan variance is used rather than Allan variance, and its equation is:

$$\sigma_y(\tau) = \sqrt{\frac{\sum_{i=1}^{n-1} (x_{i+1} - x_i)^2}{2(n-1)}}$$

Where:

- y = *fractional frequency deviation*
- τ = *measurement time for each frequency reading*
- x_n = *frequency reading*
- n = *number of frequency readings*

Statistical rules apply, and at least 31 readings (corresponding to 30 successive pairs) should be taken. Also, there will be a distinct root Allan variance value for each gate time used to record the frequency data. An example of a root Allan variance specification might be: $< 3\text{x}10^{-9}$/1 second. This states that the standard deviation of successive readings taken with 1 second gate times (not counting aging and ambient effects) should be less than $3\text{x}10^{-9}$ (fractional frequency movement).

Note that there is a direct (albeit complex) relation between Allan variance and phase noise ($\mathcal{L}(f)$): Longer gate times used in measuring Allan variance correspond to smaller frequency offsets from the carrier, while shorter gate times correspond to larger frequency offsets from the carrier. The phase noise measurement system used has the capability to calculate root Allan variance from $\mathcal{L}(f)$. This is the normal method Isotemp uses to characterize the root Allan variance performance of an oscillator for measurement times $\leq$10 seconds.

Refer to figures 1-3 for a graphical example. Figure 1 shows a ’continuous’ Frequency vs Time curve for a hypothetical oscillator. Notice that long term drift has been added to simulate aging or ambient effects. Figure 2 shows the average of the continuum for periods of 1 second in length. The real world equivalent would be taking 10 frequency readings with a counter set to a 1 second gate time. There are 9 successive frequency pairs with which to perform the root Allan variance calculation. Figure 3 is similar to figure 2, except the averaging period has been doubled to 2 seconds. There would now only be 5 frequency readings, with 4 pairs of data.

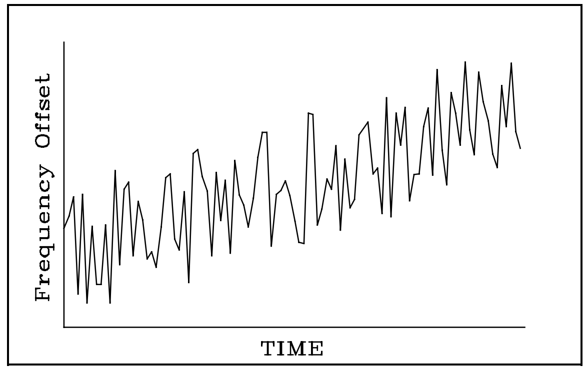

Figure 1

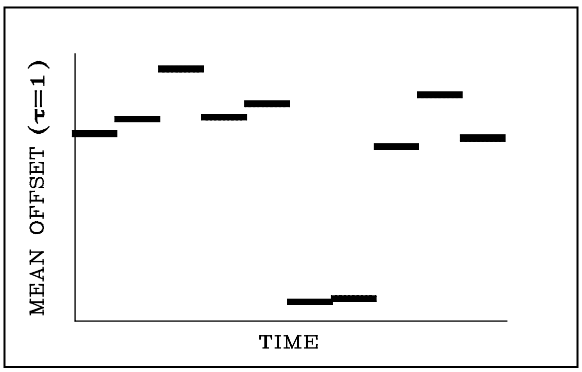

Figure 2

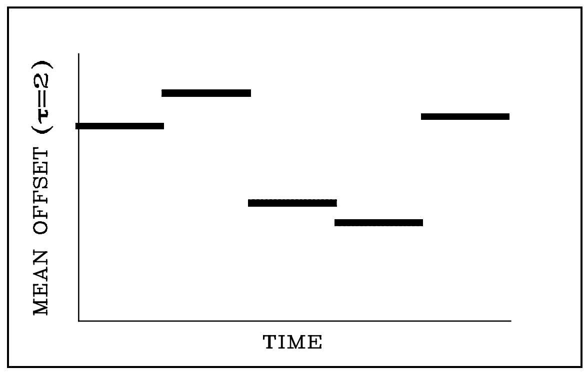

Figure 3

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